

Primordial bubbles of colour superconducting quark matter

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Abstract

We analyze the possibility that bubbles of quark matter surviving the confinement phase transition might have become colour superconducting due to the enormous compression suffered by them.

Because of the relatively high temperature of the process we compare the initially unpaired quark matter with the colour-flavour locked alternative when the extremely large chemical potential could have increased the critical temperature sufficiently and find that this latter phase would be more stable before the bubble compression stops.

If other physical effects have not affected completely their stability, these bubbles might still exist today and perhaps be observed as strangelets.

1 Introduction

New phases of QCD are being intensively studied with the possibility of future phenomenological evidence both from accelerators and astrophysical observations.

Apart from the ordinary hadronic phase and the expected deconfined quark-gluon plasma (QGP) at high temperature, interesting colour superconducting states of quark matter are theoretically predicted at very high pressure[1][2].

There is agreement that the ultimate stable phase at extremely large chemical potential μ , and consequently pressure, is the colour flavour locked one (CFL) where couples with zero total momentum of quarks of different colours and flavours $u d s$ are paired due to the fact that mass m_s is no longer relevant. For decreasing chemical potential, before reaching the ordinary nuclear matter, it is possible that intermediate phases prevail such as colour crystals[3] due to pairing of quarks of different momentum, or Cooper pairs of only light quarks $u d$ ($2SC$), or even single flavour diquark condensates[4].

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During the cooling of universe, when the temperature reached $T \sim 150 \text{ MeV}$ presumably a first-order phase transition occurred[5], even though there is debate on this point, coincident with breaking of the approximate chiral symmetry, passing from deconfined almost free quarks in thermodynamic equilibrium to quarks confined in hadrons.

If the transition was of first order, it was produced by the expansion of bubbles of low-temperature phase in the sea of the high-temperature one. But when the coalescence velocity became smaller than that of the expansion of universe, a reversed situation occurred, with bubbles of the surviving quark matter surrounded by the more stable hadronic phase[6].

The negative pressure due to the energy of the false vacuum[7] inside these bubbles makes them contract. The chemical potential which remained constant in both phases during the transition begins to increase inside the bubble assuming no emission of baryons. This increase is not very relevant during the first stage of the compression when quarks and antiquarks annihilate with emission of neutrinos because of their very long mean free path. But in the final stage, when almost all antiquarks have disappeared due to the matter-antimatter asymmetry generated by earlier processes in the universe, the chemical potential increase makes the quark pressure rise rapidly and the bubble mass starts to increase too. Therefore the total pressure becomes positive and tends to decrease the contraction velocity.

The interval of time of the bubble contraction turns out to be small compared to the age $\sim 10^{-5}$ sec of the universe at the confinement transition. Therefore the temperature may be considered constant and must be compared with the critical temperature for the loss of colour superconductivity. For $T \sim 150 \text{ MeV}$ of all the superconducting phases only the *CFL* one can be probably reached at extremely high chemical potential. This occurs when the bubble has not yet stopped its contraction and from this moment the *CFL* free energy for variable number of particles can be evaluated as the sum of a free-quark contribution with the constraint of equal density for the 3 flavours, which is slightly larger than the unpaired value, minus a term due to Cooper pairs gap which makes the superconducting state more stable. Since this free energy $\frac{f}{V} = -p$, the pressure increases even faster and the bubble contraction is very quickly stopped.

In Sec.2 we give the thermodynamic expressions for massless and massive quarks of flavour and energy densities and of pressure as functions of temperature and chemical potential, including also a description of the *CFL* phase. Sec.3 is devoted to the calculation of the bubble contraction through the evolution of chemical potentials which determine its mass, pressure and velocity. Finally in Sec.4 we comment on physical aspects which should be also taken into account and on the possibility that a part of these bubbles might have survived till our times and could be detected by high-altitude observatories.

2 Thermodynamics of free and paired quarks

For the deconfined *QGP* phase the high T allows the approximation of neglecting the interaction among quarks so that for each fermion of mass m , with 2 spin projections and 3 colours, the number and energy densities and the pressure are given by

$$\frac{N}{V} = \frac{3}{\pi^2} \int_m^\infty d\varepsilon \varepsilon \sqrt{\varepsilon^2 - m^2} \frac{1}{\exp\left[\frac{1}{T}(\varepsilon - \mu)\right] + 1} \quad , \quad (a) \quad (1)$$

$$\frac{E}{V} = \frac{3}{\pi^2} \int_m^\infty d\varepsilon \varepsilon^2 \sqrt{\varepsilon^2 - m^2} \frac{1}{\exp\left[\frac{1}{T}(\varepsilon - \mu)\right] + 1} \quad , \quad (b)$$

$$p = \frac{1}{\pi^2} \int_m^\infty d\varepsilon (\varepsilon^2 - m^2)^{\frac{3}{2}} \frac{1}{\exp\left[\frac{1}{T}(\varepsilon - \mu)\right] + 1} \quad . \quad (c)$$

Expanding in powers of $\frac{m^2}{\varepsilon^2}$ and calling $a = \frac{m}{T}$, $x = \frac{\mu}{T}$,

$$\frac{\pi^2}{3T^3} \frac{N}{V} \simeq \exp x \int_{\exp a}^{\exp \Lambda} \frac{d\eta}{\eta(\eta + \exp x)} \left(\ln^2 \eta - \frac{a^2}{2} - \frac{a^4}{8} \frac{1}{\ln^2 \eta} \right) \quad , \quad (a) \quad (2)$$

$$\frac{\pi^2}{3T^4} \frac{E}{V} \simeq \exp x \int_{\exp a}^{\exp \Lambda} \frac{d\eta}{\eta(\eta + \exp x)} \left(\ln^3 \eta - \frac{a^2}{2} \ln \eta - \frac{a^4}{8} \frac{1}{\ln \eta} \right) \quad , \quad (b)$$

$$\frac{\pi^2}{T^4} p \simeq \exp x \int_{\exp a}^{\exp \Lambda} \frac{d\eta}{\eta(\eta + \exp x)} \left(\ln^3 \eta - \frac{3}{2} a^2 \ln \eta + \frac{3}{8} \frac{a^4}{\ln \eta} \right) \quad , \quad (c)$$

in terms of a cut-off Λ which will not appear in the final results.

We will consider quarks u and d with $a \simeq 0$ and quark s with $a \simeq 1$ at the confinement transition T .

If $x < a$, which will happen for s at the beginning of bubble contraction and for all antiquarks because $\mu_{\bar{q}} < 0$, we may expand the denominators in powers of $\frac{\exp x}{\eta}$ to obtain

$$\begin{aligned} \frac{\pi^2}{3T^3} \frac{N}{V} \simeq & \exp(x - a) \left(2 + 2a + \frac{a^2}{2} - \frac{a^3}{8} \right) - \\ & - \frac{\exp[2(x - a)]}{2} \left(\frac{1}{2} + a + \frac{a^2}{2} - \frac{a^3}{4} \right) + \\ & + \frac{\exp[3(x - a)]}{3} \left(\frac{2}{9} + \frac{2}{3}a + \frac{a^2}{2} - \frac{3}{8}a^3 \right) - \\ & - \frac{a^4}{8} [\exp(x) Ei(-a) - 2 \exp(2x) Ei(-2a) + 3 \exp(3x) Ei(-3a)] \quad , \end{aligned} \quad (a) \quad (3)$$

$$\begin{aligned}
\frac{\pi^2}{3T^4} \frac{E}{V} \simeq & \exp(x-a) \left(6 + 6a + \frac{5}{2}a^2 + \frac{a^3}{2} \right) - \\
& - \frac{\exp[2(x-a)]}{2} \left(\frac{3}{4} + \frac{3}{2}a + \frac{5}{4}a^2 + \frac{a^3}{2} \right) + \\
& + \frac{\exp[3(x-a)]}{3} \left(\frac{2}{9} + \frac{2}{3}a + \frac{5}{6}a^2 + \frac{a^3}{2} \right) + \\
& + \frac{a^4}{8} [\exp(x) Ei(-a) - \exp(2x) Ei(-2a) + \exp(3x) Ei(-3a)] ,
\end{aligned} \tag{b}$$

$$\begin{aligned}
\frac{\pi^2}{T^4} p \simeq & \exp(x-a) \left(6 + 6a + \frac{3}{2}a^2 - \frac{a^3}{2} \right) - \\
& - \frac{\exp[2(x-a)]}{2} \left(\frac{3}{4} + \frac{3}{2}a + \frac{3}{4}a^2 - \frac{a^3}{2} \right) + \\
& + \frac{\exp[3(x-a)]}{3} \left(\frac{2}{9} + \frac{2}{3}a + \frac{a^2}{2} - \frac{a^3}{2} \right) - \\
& - \frac{3}{8}a^4 [\exp(x) Ei(-a) - \exp(2x) Ei(-2a) + \exp(3x) Ei(-3a)] .
\end{aligned} \tag{c}$$

On the other hand for $x > a$, which is needed for u and d and occurs for s at the end of the bubble contraction, the integrals of Eq.(2) will be the sum of two parts one of the same type but with lower limit $\exp x$ to allow the expansion in powers of $\frac{\exp x}{\eta}$ and another between $\exp a$ and $\exp x$ where the expansion of the denominator is in powers of $\frac{\eta}{\exp x}$. The results are

$$\begin{aligned}
\frac{\pi^2}{3T^3} \frac{N}{V} \simeq & \frac{x^3}{3} + \frac{x^2}{3} + \frac{29}{9}x + \frac{2}{27} - \frac{1}{2}a^2x - \\
& - \frac{1}{6}a^2 + \frac{a^3}{24} + \exp[-(x-a)] \left(2 - 2a + \frac{a^2}{2} + \frac{a^3}{8} \right) - \\
& - \frac{\exp[-2(x-a)]}{2} \left(\frac{1}{2} - a + \frac{a^2}{2} + \frac{a^3}{4} \right) + \frac{a^4}{8} [-\exp(x) Ei(-x) + \\
& + 2\exp(2x) Ei(-2x) - 3\exp(3x) Ei(-3x) + \exp(-x) Ei(x) - \\
& - 2\exp(-2x) Ei(2x) - \exp(-x) Ei(a) + 2\exp(-2x) Ei(2a)] ,
\end{aligned} \tag{a} \tag{4}$$

$$\begin{aligned}
\frac{\pi^2}{3T^4} \frac{E}{V} \simeq & \frac{x^4}{4} + \frac{x^3}{3} + \frac{29}{6}x^2 + \frac{2}{9}x + \frac{1223}{108} - \frac{1}{4}a^2x^2 - \\
& - \frac{1}{6}a^2x - \frac{29}{36}a^2 + \exp[-(x-a)] \left(-6 + 6a - \frac{5}{2}a^2 + \frac{a^3}{2} \right) + \\
& + \frac{\exp[-2(x-a)]}{2} \left(\frac{3}{4} - \frac{3}{2}a + \frac{5}{4}a^2 - \frac{a^3}{2} \right) + \frac{a^4}{8} [\exp(x) Ei(-x) - \\
& - \exp(2x) Ei(-2x) + \exp(3x) Ei(-3x) + \exp(-x) Ei(x) - \\
& - \exp(-2x) Ei(2x) - \exp(-x) Ei(a) + \\
& + \exp(-2x) Ei(2a) - \ln x + \ln a] ,
\end{aligned} \tag{b}$$

$$\begin{aligned}
\frac{\pi^2}{T^4} p \simeq & \frac{x^4}{4} + \frac{x^3}{3} + \frac{29}{6}x^2 + \frac{2}{9}x + \frac{1223}{108} + \frac{a^4}{2} - \quad (c) \\
& - \frac{3}{2}a^2 \left(\frac{x^2}{2} + \frac{x}{3} + \frac{29}{18} \right) + \exp[-(x-a)] \left(-6 + 6a - \frac{3}{2}a^2 - \frac{a^3}{2} \right) + \\
& + \frac{\exp[-2(x-a)]}{2} \left(\frac{3}{4} - \frac{3}{2}a + \frac{3}{4}a^2 + \frac{a^3}{2} \right) + \frac{3}{8}a^4 [-\exp(x) Ei(-x) + \\
& + \exp(2x) Ei(-2x) - \exp(3x) Ei(-3x) - \exp(-x) Ei(x) + \\
& + \exp(-2x) Ei(2x) + \exp(-x) Ei(a) - \\
& - \exp(-2x) Ei(2a) + \ln x - \ln a] .
\end{aligned}$$

To analyze the relative stability of states of systems with variable number of particles we must compare the free energy

$$\frac{J}{V} = \frac{E}{V} - T \frac{S}{V} - \mu \frac{N}{V} = -p \quad , \quad (5)$$

summed over quarks and antiquarks of all flavours.

We now turn to the possibility that quarks form Cooper pairs.

For the case where $\mu \gg m_s$ and T is not too high it seems clear that di-quark condensates are formed due to their interaction which is attractive in the colour antisymmetric multiplet $\bar{3}$, favoured in the antisymmetric zero total spin and flavour antisymmetry to allow total antisymmetry of fermions, *i.e.*

$$\langle q_{iL}^\alpha(\mathbf{p}) q_{iL}^\beta(-\mathbf{p}) \rangle = \Delta \sum_{c=1}^3 \varepsilon^{\alpha\beta c} \varepsilon_{ijc} \quad , \quad (6)$$

where α and i are respectively colour and flavour indices and the pair has the same chirality because the momenta of the partners are equal and opposite. Analogous expression holds for R chirality.

In this CFL phase both the gauge $SU(3)_c$ and the approximate global chiral $SU(3)_{L,R}$ symmetry are broken giving respectively massive gluons and mesons as Goldstone particles.

The gap Δ can be calculated in the Nambu-Jona Lasinio approximation of QCD where an effective 4-fermion interaction with coupling G is considered.

For $T = 0$ the estimation[8]

$$\Delta_0 \simeq \mu \exp\left(\frac{-\pi^2}{2G\mu^2}\right) \quad (7)$$

may be given with $G \simeq \frac{1}{G_{\text{ev}^2}}$.

More precise evaluations[2] are done with perturbative QCD with coupling g giving

$$\Delta_0 = b \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \quad , \quad (8)$$

where the coefficient is primarily $b = 512 \pi^4 \left(\frac{2}{N_f}\right)^{\frac{5}{2}}$, N_f is the number of flavours that we take 3 and $\alpha_s = \frac{g^2}{4\pi} \simeq \frac{0.7}{\ln\left(\frac{\mu}{\Lambda_{QCD}}\right)}$ with $\Lambda_{QCD} = 200 \text{ MeV}$. But additional

corrections to b may decrease the above expression by a factor of 5 or increase it by 20.

For $\mu \sim 1 \text{ GeV}$ Eq.(8) gives $\Delta_0 \sim 50 \text{ MeV}$ in agreement with the result with Eq.(7). Even though one cannot trust the extrapolation of the perturbative calculation down to $\sim 1 \text{ GeV}$, Eq.(8) shows clearly that because of the asymptotic freedom limit $g \rightarrow 0$ for $\mu \rightarrow \infty$, finally Δ_0 will decrease.

For $T > 0$ there will be a non-vanishing gap provided $T < T_c \simeq 0.57\Delta_0$. Therefore for very large μ colour superconductivity might be achieved even at the confinement temperature $T \simeq 150 \text{ MeV}$.

To evaluate the free energy J at $T = 0$ one has to sum the expression for the unpaired case but with the restriction of equal Fermi momentum for all flavours, in order to have the same number of quarks, plus a contribution coming from the gap[9], *i.e.*

$$\left. \frac{J}{V} \right|_{\text{paired}}^{(T=0)} = \left. \frac{J}{V} \right|_{\text{unpair}}^{(T=0)} (p_F^u = p_F^d = p_F^s) - \frac{3}{\pi^2} \Delta_0^2 \mu^2 \quad . \quad (9)$$

The condition of equal Fermi momenta increases J_{unpair} compared to the same expression for free quarks but the difference is compensated by the last term of Eq.(9) for large enough μ .

For $T > 0$ the corresponding formula will have J_{unpair} with chemical potentials such that the densities of quarks of different flavours are equal [10] and a contribution of gap which vanishes for a continuous transition[8] at $T = T_c$, *i.e.*

$$\left. \frac{J}{V} \right|_{\text{paired}} = \left. \frac{J}{V} \right|_{\text{unpair}} (n_u = n_d = n_s) - \frac{18\mu^2 T_c^2}{7\zeta(3)} \left(1 - \frac{T}{T_c}\right)^2 \quad . \quad (10)$$

The pressure of the paired quarks will be therefore larger than that of non-interacting quarks for T clearly below T_c .

3 Calculation of bubble contraction

We will consider the force applied to the bubble, accepting that the confinement transition was first order, causing the increase of its relativistic momentum in radial direction together with release of the momentum of neutrinos emitted due to quark-antiquark annihilation

$$F = \frac{d}{dt} \left(\frac{M}{\sqrt{1-v^2}} v \right) + \frac{dP_\nu}{dt} \quad , \quad (11)$$

where the rest mass M comes from quark, antiquark and false vacuum energy contributions. The force will result from the difference Δp between the pressure of false vacuum and that of quarks and antiquarks inside the bubble applied to its surface.

Taking the bubble radius r negative in order to have positive velocity v during contraction, from Eq.(11) we have

$$\Delta p 4\pi r^2 \sqrt{1-y} = y \frac{dM}{dr} + \frac{M}{1-y} \frac{1}{2} \frac{dy}{dr} + \frac{dP_\nu}{dt} \sqrt{1-y} \quad , \quad (12)$$

where $y = v^2$.

In the first stage of the bubble compression $\Delta p > 0$ and the rest mass decreases because quarks and antiquarks annihilate to maintain the density determined by thermodynamic equilibrium. On its side the momentum of the copiously produced neutrinos, as we will see, tends to compensate the decrease of the first term of *RHS* of Eq.(12). Therefore, the second term shows an increase of the velocity even though smaller than without the ν term.

On the other hand, when the bubble radius becomes small, almost all the antiquarks have been annihilated and the produced neutrinos momentum is negligible whereas the quark chemical potential raises rapidly the pressure so that $\Delta p < 0$ and gives also an increase of M . As a consequence the velocity decreases and finally the bubble compression stops.

In our calculation we consider the system with neutral electric charge and make the approximations of neglecting the presence of electrons and of surface effects.

The condition of zero charge density gives the baryonic density as

$$\frac{N_B}{V} = \frac{N_u - N_{\bar{u}}}{V} = \frac{1}{2} \left(\frac{N_d - N_{\bar{d}}}{V} + \frac{N_s - N_{\bar{s}}}{V} \right) \quad . \quad (13)$$

At the beginning $\mu \ll T$ because with this approximation Eq.(1a) gives

$$\frac{N_q}{V} \simeq \frac{5.46}{\pi^2} T^3 + \frac{4.92}{\pi^2} T^2 \mu \quad , \quad (14)$$

so that the contribution of quarks to baryonic density is

$$\frac{N_u - N_{\bar{u}}}{V} \simeq \frac{9.84}{\pi^2} T^2 \mu \quad . \quad (15)$$

Since the entropy density is $s \sim T^3$, the normalized matter-antimatter asymmetry necessary for the primordial nucleosynthesis gives

$$\frac{n_B}{s} \sim 10^{-10} \sim \frac{\mu}{T} \quad , \quad (16)$$

determining for the transition temperature $\sim 100 \text{ MeV}$ the consistently small value of chemical potential $\mu \sim 10^{-2} eV$.

If the bubble suffers no evaporation it will conserve the baryonic number during the contraction so that, being [6] the initial radius $\sim 1 \text{ cm}$, according to Eq.(15) μ will increase reaching T for a radius $\sim 10^{-3} \text{ cm}$.

Initially the bubble will be compressed by an "external pressure" given by the negative of the energy density of the false vacuum $\rho_V \simeq \Lambda_{QCD}^4$. But when μ

Figure 1: Rest mass M of bubble including quark-antiquark thermodynamic contributions at $T = 150 \text{ MeV}$ and false vacuum energy.

surmounts T and practically no more antiquarks survive, according to Eq.(4c) the pressure of the 3 flavours may be estimated by $\sim \frac{3}{4\pi^2}\mu^4$ which will overcome the external one and Δp of Eq.(12) will change its sign.

Similarly, at the beginning when μ is very small the rest mass of the bubble is given by an almost constant energy density according to Eqs.(3b,4b) plus the constant false vacuum energy density Λ_{QCD}^4 , so that M will decrease with V . But when $\mu > T$ the term $\sim \mu^4$ in the energy density will dominate making M increase with the bubble contraction.

Finally, taking the effective theory as indicative for the values of chemical potential achieved in the process, according to Eq.(7) when $\mu \gtrsim 10 T$ the gap Δ_0 may be sufficiently large to allow $T < T_c$ and make the bubble colour superconducting before it stops.

The numerical procedure for the evaluation of Eq.(12) has been as follows. We started with a radius variable $r_0 = -1 \text{ cm}$ and zero velocity $v_0 = 0$. For the different steps the conservation of baryonic number N_B gave through Eq.(13) the chemical potential μ_u and a relation between μ_d and μ_s . These two latter chemical potentials were determined by minimizing the energy at each radius. Then the pressure given by quarks and antiquarks could be calculated. The precise value of the negative pressure of false vacuum was adjusted to be slightly larger than that of quarks and antiquarks in the initial stage in order to have the minimal compression condition.

Regarding the inclusion of the momentum of neutrinos in Eq.(11) it is convenient to consider the two stages of $\mu \ll T$ and $\mu \gtrsim T$.

In the former we may approximate the energy density as a constant $\rho =$

Figure 2: Bubble chemical potential and internal pressure. Continuous line $\log \mu_u [MeV]$, broken line $\log(\frac{\pi^2 p}{T^4})$.

$\rho_V + \rho_{q,\bar{q}}$ where $\rho_{q,\bar{q}}$ is due to all quarks and antiquarks, so that $\frac{dM}{dr} = -\rho \cdot 4\pi r^2$. In the rest frame $\rho_{q,\bar{q}}$ is transformed into neutrinos which will be isotropically distributed with an average momentum k such that

$$dE_\nu = \rho_{q,\bar{q}} 4\pi r^2 v dt = 4\pi k^2 \sigma \quad , \quad (17)$$

where σ is surface energy density in space \mathbf{k} .

Taking the motion in the direction 3, the momentum seen from lab frame will be $k'_3 = \frac{k \cos \theta + v k}{\sqrt{1-v^2}}$. Therefore there will be a total momentum in lab system

$$dP_3 = k^2 \int \left(\frac{\sigma}{k} \right) k'_3 \sin \theta d\theta d\varphi = \frac{v^2}{\sqrt{1-v^2}} \rho_{q,\bar{q}} 4\pi r^2 dt \quad , \quad (18)$$

and the neutrino momentum will cancel the contribution of $\rho_{q,\bar{q}}$ in $\frac{dM}{dt}$ of Eq.(12), leaving

$$\Delta p \sqrt{1-y} = -\rho_V y + \frac{1}{6} |r| \rho \frac{1}{1-y} \frac{dy}{dr} \quad . \quad (19)$$

In the small radius stage when $x \gtrsim 1$, the antiquarks have been almost all annihilated so that $\rho_{\bar{q}} \ll \rho_q$ and only $2\rho_{\bar{q}}$ will transform into neutrinos. Therefore Eq.(12) must be used with

$$\sqrt{1-y} \frac{dP_\nu}{dt} = 4\pi r^2 y 2\rho_{\bar{q}} \quad . \quad (20)$$

In this last part of bubble compression, when $T < T_c$ the chemical potentials μ_d and μ_s will be determined by the condition of having equal density of u , d and s . The pressure of quark matter will be increased by the gap term of Eq.(10).

Figure 3: Velocity of contraction of surviving quark matter bubble at $T = 150 \text{ MeV}$.

In this way the properties of the bubble were calculated, *i.e.* the rest mass M (Fig.1) the chemical potential and the internal pressure p (Fig.2) and the velocity of contraction v (Fig.3).

From them it is seen that the bubble stops its compression for a final radius R_f slightly below 10^{-4} cm , its rest mass starts to increase for radius slightly below 10^{-3} cm when also the increase of the internal pressure overcomes the external one, and in between these sizes μ reaches a value $\sim 1.5 \text{ GeV}$ so that $T < T_c$ allowing the possibility that the bubble quark matter becomes colour superconducting.

Finally, the interval of time for the bubble contraction

$$\Delta t = \int_{r_0}^{r_f} \frac{dr}{v} \quad , \quad (21)$$

can be calculated numerically giving $\Delta t \sim 10^{-10} \text{ sec}$ much smaller than 10^{-5} sec so that the approximation of $T \sim \text{constant}$ is consistent.

4 Conclusions

According to our model, the bubbles of quark matter surviving the confinement transition may reach such high values of chemical potential, larger than those in neutron stars, that could enter the colour superconducting *CFL* phase before their contraction stops becoming therefore stable. A more accurate analysis of the order of the transition between *QGP* and *CFL* phases[11] should be done to establish with certainty the dependence of T_c on μ .

There are several physical considerations that may affect the above result. One is the emission of neutrons during the bubble compression which would make the increase of chemical potential softer. Another thing is the possible inclusion of a density of electrons which would alter the quark chemical potentials to satisfy charge neutrality. Finally, taking into account surface effects the condition of charge neutrality of the bubble may not be realistic.

In case that the bubbles, perhaps smaller, can nevertheless become colour superconducting at the relatively large temperature $\sim 150 \text{ MeV}$, one has to take into account that if their energy per baryon is larger than 940 MeV they will emit neutrons during the cooling till the present time. If this process leaves bubbles of atomic number $A > 10^3$ at $T \sim 0$ they would be absolutely stable[12] and might be detected as strangelets by high-altitude observatories since the collisions in the upper part of the atmosphere would not reduce substantially their mass maintaining therefore their stability.

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